Nuclear Structure of ^{6}Li and High-Energy Electron Scattering *

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The nuclear charge form factgr from the high-energy elastic electron scattering on ^{6}Li has been calculated from the modified independent-particle shell model (IPSM) wave function. The usual harmonic oscillator type IPSM wave function has been modified by the inclusion of a nucleon-nucleon correlation function which involves extra-core nucleons only. The technique is extremely simple and provides an excellent agreement with the experimental data.

High-energy electron scattering has become a powerful method for studying the structure of atomic nuclei. It is well known that, except for ^{6}Li, the nuclear form factors in the 1p shell, as obtained from elastic electron scattering can be derived from ordinary single particle harmonic oscillator wave functions. SUELZLE et al. [1] published some excellent experimental data on elastic scattering from lithium isotopes. The form factor obtained from the ordinary harmonic oscillator wave function agrees with the data for low momentum transfer. However, in the region of large momentum transfer the fit is quite inadequate. The usual shell-model wave function of ^{6}Li with a 1s closed shell and two 1p nucleons is a Slater determinant of single-particle functions determined in a central potential well. The breakdown of such a rudimentary wave function in the region of high-momentum transfer is perhaps due to the fewer high-momentum components in this wave function. One can introduce the high-momentum components by introducing the short range as well as long range potentials in the nucleon-nucleon potential. Alternatively, the velocity-dependent potential can introduce high-momentum components in the wave function [2]. To modify the wave function in the former manner is to do the Brueckner-type calculations [3] for this finite nuclear system. Such an approach is extremely difficult and gives rise to many uncertainties. We propose here a somewhat phenomenological method which nevertheless retains all the qualitative features of the more fundamental approach and is, at the same time, relatively simple.

In case of ^{6}Li the basic assumption of the IPSM – that the motion of a particle is independent of the other particles – is not quite likely to be valid. Particularly while studying the electromagnetic properties of a nucleon inside a nucleus one should give serious considerations to the effects of the near-by nucleons on its behavior. Here we consider a simple model of ^{6}Li in which the four 1s particles form a doubly closed shell having some statical nucleon-nucleon correlation due to Pauli exclusion principle whereas the two valence nucleons in 1p shell are treated somewhat more explicitly. A dynamical nucleon-nucleon correlation is also invoked for these particles in addition to the Pauli correlation. The dynamical correlation existing among these nucleons can be understood naively as reflecting a nonlocal potential depending on the relative coordinate $(\boldsymbol{r}_1-\boldsymbol{r}_2)$ in the ordinary space. The modified wave function, Ψ, for ^{6}Li is chosen to have the symbolic form

$$\Psi=\frac{1}{N}\,\psi(\text{core})\,\psi(1,2)\,\chi(1,2,3,4,5,6) \qquad (1)$$

where χ represents the spin and i-spin parts of the total wave function and $\psi(1,2)$ is the correlated part of the wave function of the two 1p nucleons given by

$$\psi(1,2)=\psi^0(1,2)\,(1+f_{ij})\,. \qquad (2)$$

The single-particle harmonic oscillator type wave functions $\psi(\text{core})$ and $\psi^0(1,2)$ refer to the α-particle core and the two 1p nucleons group, respectively. The normalization constant N is given by

$$N^2=1+2\langle f_{ij}\rangle+\langle f_{ij}{}^2\rangle\,. \qquad (3)$$

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The wave function (1) still gives the proper anti-symmetric properties. It is multiplied by its appropriate spin and iso-spin functions as demanded by the Pauli exclusion principle. It may also be noted that (1) is also an eigen function of L^2, L_z, S^2 and S_z (see note [4]). The two body correlation factor f_{ij} is a phenomenological type of simple function chosen to be $C\,r_{12}$ for these calculations. The correlation coefficient C is an adjustable parameter which is found to have a negative value always. Most of the authors have invoked the two-body correlation functions in the exponentially decaying forms. In that spirit the correlation function used here may be regarded as the first two dominant terms of the exponential expansion. It may be remarked that the motivation of using the correlation function $(1+C\,r_{12})$ in this work is not the same as it might appear from the preceeding sentence. It may, however, be interpreted as a compromise between the type used here and the other exponential types employed by other authors. The expansion of r_{12} in the Legendre polynomials gives rise to the following expression

$$r_{12}=\sum_{k=0}^{\infty} U_k(r_<, r_>)\, P_k(\cos\omega_{12}), \qquad (4)$$

where $r_<$ ($r_>$) is the smaller (greater) of r_1 and r_2, and ω_{12} is the angle between the vectors $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$. The expansion of U_k is given explicitly as

$$U_k(r_1, r_2) = \frac{r_1^{k+2}}{(2\,k+3)\; r_2^{k+1}} - \frac{r_1^{k}}{(1-2\,k)\; r_2^{k-1}}. \qquad (5)$$

The polynomial (5) of the arguments r_1 and r_2 are similar to those which appear in the single-particle harmonic oscillator type radial wave functions for higher orbitals while mixing the higher configurations [5]. Although the admixture of many particle states is well founded in theory, it is rather cumbersome to handle. It is desirable to replace the admixture of higher configurations by a convenient two-body correlation function $(1+C\,r_{12})$. The latter is simulated to incorporate all the features which the former does. This type of correlated wave function expediently yields interesting results when applied to nuclear problems and is convenient to handle despite the inclusion of relative coordinates in the independent-particle wave function. The correlated wave function of ^{6}Li used here assumes definitely the alpha and deuteron clustering effect in this nucleus which is well supported by several experimental data and other calculations. In this representation there are two main approximations. The exchange degeneracy and spatial correlation between the 1s and 1p nucleons and also among the 1s nucleons are ignored. At the realistic isolation of the two groups of nucleons the exchange effects are of no practical importance [6, 7]. It was shown earlier that the exchange effect of the s- and p-particles is negligibly small on the rms radius of ^{6}Li [8]. Regarding the correlation effect it has been recently shown in the electron scattering calculations for ^{6}Li that an appreciable part of the total correlation effect is derived from particles in other than relative S-states [9]. In this note attempt is made to understand correlation phenomenon while studying the elastic electron scattering by ^{6}Li in a simple minded manner. In the L-S coupling, which is quite suitable for ^{6}Li, we have for the ground state

$$L=0, \quad S=1, \quad J=1, \quad T=0.$$

In the Born approximation the elastic electron scattering cross section for electrons of energy E on a nucleus with atomic number Z and mass M is well known to have the form [10]

$$\frac{d\sigma}{d\Omega}=\frac{Z^2\,e^4\cos^2(\Theta/2)}{4\,E^2\sin^4(\Theta/2)}\;\frac{|F(q)|^2}{1+(2\,E/M)\sin^2(\Theta/2)}, \qquad (6)$$

where Θ is the scattering angle, and the units are such that $\hbar=c=1$. $F(q)$ is the charge form factor of the target nucleus as obtained from elastic electron scattering and is given by [11]

$$F(q) = \int \exp\{i\,\boldsymbol{q}\cdot\boldsymbol{r}\}\,\varrho(r)\,dr, \qquad (7)$$

where

$$\varrho(\boldsymbol{r}_i) = \frac{1}{Z}\sum_{i=1}^{Z}\int |\Psi|^2\, d\boldsymbol{r}_1\, d\boldsymbol{r}_2, \ldots, d\boldsymbol{r}_{i-1}\, d\boldsymbol{r}_{i+1}, \ldots, d\boldsymbol{r}_A, \qquad (8)$$

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and q is the momentum transfer. Using standard Racah algebra, the Wigner-Eckart theorem and some properties of irreducible tensor operators the matrix elements which appear in (7) can be easily evaluated.

For a light nucleus like this the center of mass motion must be taken into account. This correction is cancelled out with the correction due to the finite size of the proton [12]. Hence the final form of the form factor (7) is given by

$$F(q) = \tfrac{1}{3}\Big\{2\langle (1s)^2 | \exp\{-i\,\boldsymbol{q}\cdot\boldsymbol{r}\} | (1s)^2\rangle + \frac{1}{N^2}\big\{\langle (1p)^2 | \exp\{-i\,\boldsymbol{q}\cdot\boldsymbol{r}\} | (1p)^2\rangle + 2\langle (1p)^2 | \exp\{-i\,\boldsymbol{q}\cdot\boldsymbol{r}\} f_{ij} | (1p)^2\rangle + \langle (1p)^2 | \exp\{-i\,\boldsymbol{q}\cdot\boldsymbol{r}\}\, f_{ij}^2 | (1p)^2\rangle\big\}\Big\} \tag{9}$$

where

$$N^2 = 1 + 2\langle (1p)^2 | f_{ij} | (1p)^2\rangle + \langle (1p)^2 | f_{ij}^2 | (1p)^2\rangle, \tag{10}$$

$$= 1 + \frac{5\sqrt{2}}{\sqrt{\pi}}\frac{C}{\alpha} + \frac{5\,C^2}{\alpha^2}. \tag{11}$$

After performing the lengthy integrations but fairly straight forward Eqs. (9) and (10) immediately yield

$$F(q) = \tfrac{1}{3}\exp\{-q^2/4\,\alpha^2\}\Big\{2 + \frac{1}{N^2}\Big[(1 - q^2/6\,\alpha^2) - \sqrt{\frac{2}{\pi}}\,\frac{C}{12\,\alpha}\exp\{q^2/8\,\alpha^2\}\Big(4 + \sum_{n=1}^{\infty}\frac{(3+2\,n)^2}{2^n}\,{}_1F_1\,(-n-1;\,\tfrac{3}{2}\,;\,q^2/8\,\alpha^2)\Big) + \frac{C^2}{3\,\alpha^2}\Big(15 - \frac{15\,q^2}{4\,\alpha^2} + \frac{q^4}{8\,\alpha^4}\Big)\Big]\Big\}. \tag{12}$$

In Figure 1 the ^{6}Li form factor obtained from these calculations is compared with the experimental data [1, 13, 14]. The theoretical curve (full line) is in the excellent agreement with the experimental points taken from three experiments. The search for the best fit is made by varying the harmonic oscillator length parameter α and the correlation coefficients C. The best fit as shown in Figure 1 is obtained for $\alpha = 0.603\ \mathrm{fm}^{-1}$ and $C = -0.32\ \mathrm{fm}^{-1}$. The length parameter α used for both 1s and 1p nucleons is the same. Obviously because of the Pauli exclusion principle r_{12} does not exactly become zero, that is the two spatially correlated particles never overlap completely. However, as r_{12} increases, the spatial correlation effect goes on diminishing and ultimately

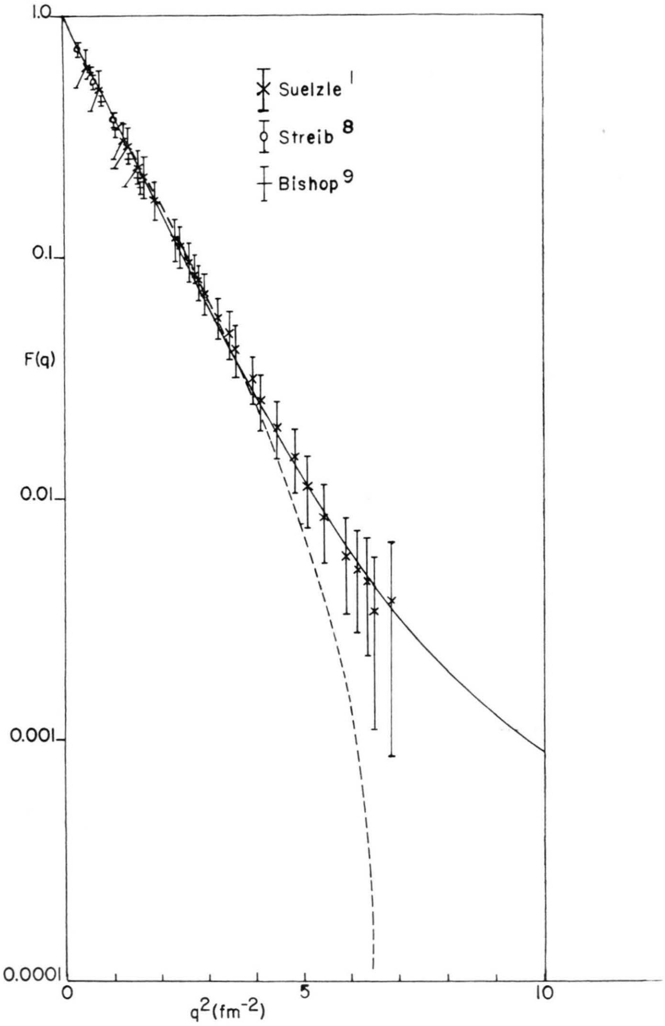

Fig. 1. ^{6}Li form factor obtained from the elastic scattering of high energy electrons. The broken line is obtained with the usual IPSM uncorrelated wave function whereas the full line is obtained with the modified correlated wave function for $C = -0.32\ \mathrm{fm}^{-1}$. Both form factors are calculated with the same oscillator parameter $\alpha = 0.603\ \mathrm{fm}^{-1}$ in Born approximation.

the total wave function (1) approaches zero as the separation coordinate r_{12} becomes approximately 3.1 fm. This zero value for the total wave function corresponds to the value $C = -0.32\ \mathrm{fm}^{-1}$ is obtained from this analysis. At this value of r_{12} and greater the ^{6}Li wave function in the bound state becomes unrealistic as one can instantly realize that there is no practical use of spreading it out any further.

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The modification of independent-particle shell model wave functions by the inclusion of the nucleon-nucleon correlation function [15] is found frequently in the current literature giving quite interesting results. However, the correlation function describing the relative motion of two nucleons, of the type used in this analysis to modify the usual shell model wave function of ^{6}Li, is much simpler. Furthermore, the correlation function involving the extra-core nucleons only (although the core nucleons do have some correlation due to the Pauli exclusion principle) makes the calculations still easier. It is established that modification of the independent-particle wave function by including some nucleon correlation function in it is important particularly for ^{6}Li in which the residual two-body interaction is significant [5] (see Figure 1 to compare the uncorrelated form factor with the correlated one). Another interesting piece of information from this analysis of the correlated wave functions which one can infer concerns the nucleon-nucleon correlation itself. One generally expects that inelastic electron scattering (compared to elastic electron scattering) can only provide information on the nucleon-nucleon correlation. However, the present analysis suggests that the elastic electron scattering too can be used as an efficient tool for this type of study.

To summarize, the correlated wave function seems to provide an excellent description for the elastic electron scattering data at all momentum transfers by ^{6}Li.

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